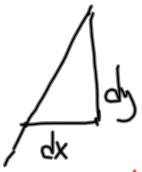
3.8/41 $\quad y=\sqrt{3x-2}\quad ; \quad x: 2 \mapsto 2.03$

Goal: to use dy to approx. Δy

$m = \frac{dy}{dx}$ at any x



$$\frac{dy}{dx} = \frac{d}{dx}(3x-2)^{\frac{1}{2}} = \frac{1}{2}(3x-2)^{-\frac{1}{2}}(3)$$

$$= \frac{3}{2\sqrt{3x-2}}$$

$$\therefore dy = \frac{3}{2\sqrt{3x-2}}\,dx$$

at $x=2$

$$dy = \frac{3}{2\sqrt{3(2)-2}}\,dx = \frac{3}{4}\,dx$$

in this problem, $\Delta x = dx = 2.03 - 2 = .03$

$$\therefore \underset{\text{(actual change)}}{\Delta y} \approx dy = \frac{3}{4}(.03) = .0225$$

$\Delta y \overset{\text{exactly}}{=} \sqrt{3(2.03)-2} - \sqrt{3(2)-2} \approx .0223748\ldots$

3.8/37)

a) $y = 4x^3 - 7x^2$

$$\frac{dy}{dx} = 12x^2 - 14x$$

so $dy = (12x^2 - 14x)\,dx = 2x(6x-7)\,dx$

b) $y = x\cos x$

$$\frac{dy}{dx} = (1)(\cos x) + (x)(-\sin x)$$

so

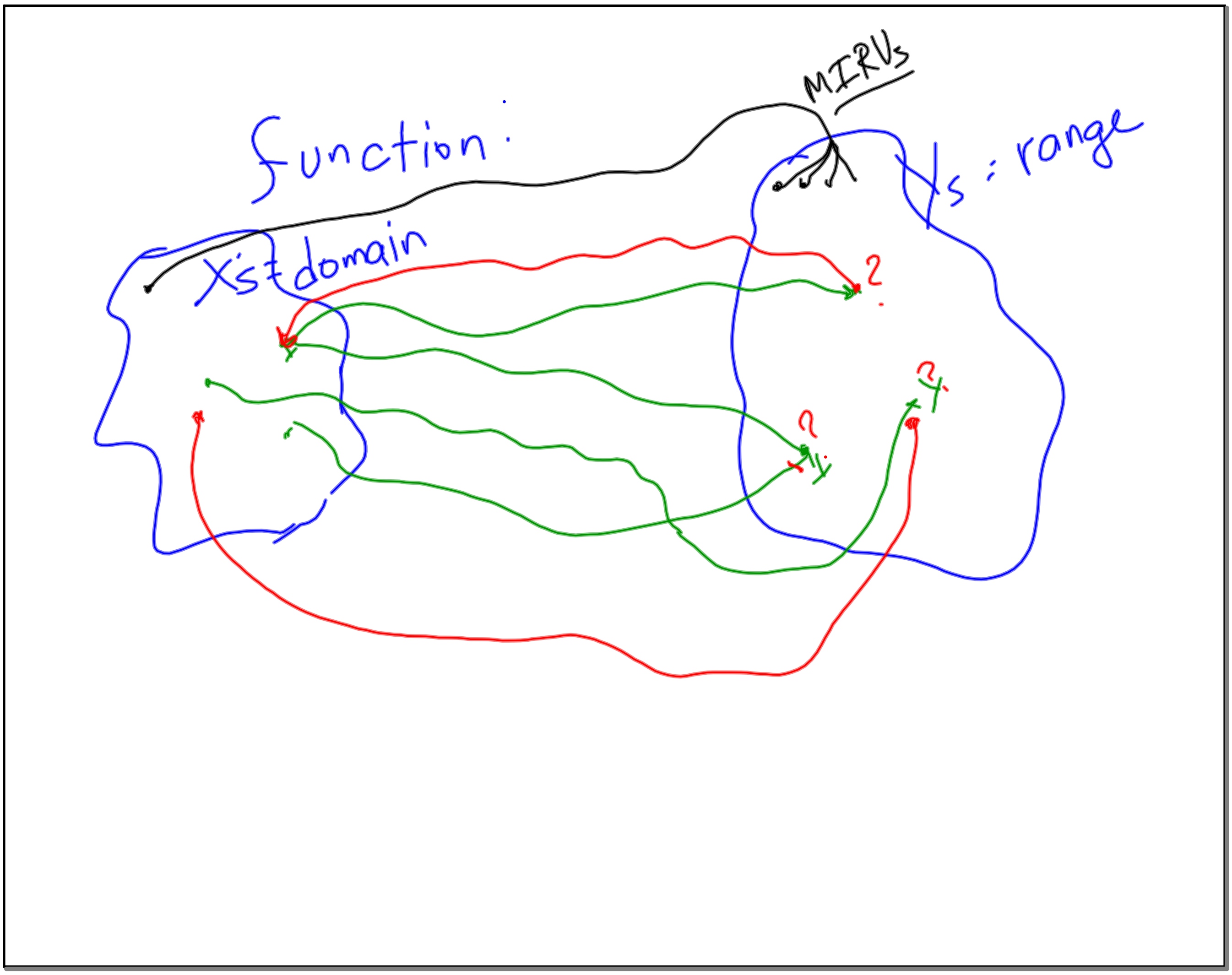
$dy = (\cos x - x\sin x)\,dx$

Oct 19-7:36 AM

4.1) Inverse Functions

One of the goals of mathematics is to identify recurring patterns [in algebra and calculus this leads to FUNCTIONS] and study their properties.

Another important (predictive) skill is to learn how to do un-do things.



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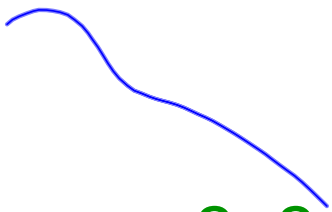
So: Question: when does a function have an inverse?

[when is a function invertible?]

Related: how do I carve up a function so a piece (subset) is invertible?



2+2-little red riding hood

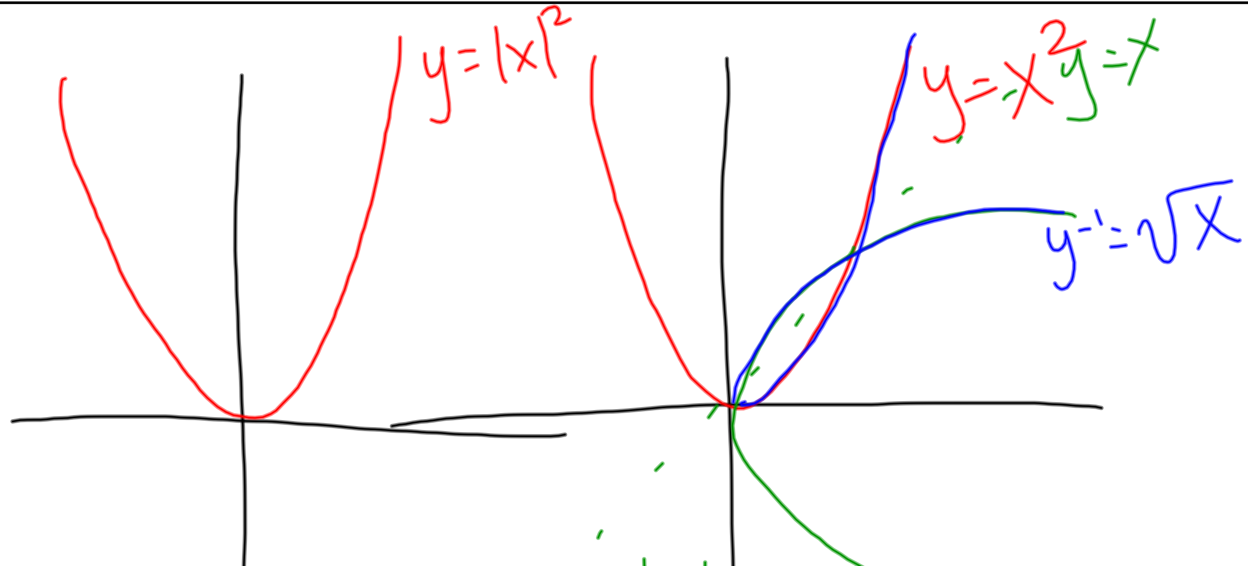
$$\lim_{x \to 0} \frac{x}{x} = 1$$

Wenbinisadoofus

Oct 19-8:17 AM

A function is invertible if it is 1-1.

A 1-1 [one to one] f^n is a function where every y-value comes from exactly 1 x.



$y=x^2$ does not have an inverse

if we restrict the domain of $y=x^2$

to $x \geq 0$

that DOES have an inverse.

inverse is $y=\sqrt{x}$

Function Properties that guarantee invertibility

(strictly)
— increasing
or
(strictly)
— decreasing

So:

A definition

A function $f(x)$ has an inverse $f^{-1}(x)$

$\iff$

$f(f^{-1}(x)) = x$ for every x in the domain of $f^{-1}(x)$

$f^{-1}(f(x)) = x$ for every x in the domain of $f(x)$.

Oct 19-8:37 AM

Consider $f(x) = x^2$

$g(x) = \sqrt{x}$

what is $f(g(x))$? $= x$

domain of $g(x)$: $[0, \infty)$

what is $g(f(x))$? $=$ "$\cancel{x}$" $|x|$

domain of $f(x)$: $(-\infty, \infty)$

Chain Rule!

$$f\left(f^{-1}(x)\right) = x$$

chain rule:

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$$

$$f\left(f^{-1}(x)\right) = x$$

$$f'\left(f^{-1}(x)\right) \cdot \frac{d}{dx} f^{-1}(x) = 1$$

Ah Ha! $\frac{d}{dx} f^{-1}(x) = \frac{1}{f'\left(f^{-1}(x)\right)}$

WARNING!

DANGER WILL ROBINSON!

$$f^{-1}(x) \neq \frac{1}{f(x)}$$

4.1 / 1-30